Exponential Functions

Answers to Exercises

1. \[
\frac{24^3}{6^2} = \left(\frac{4 \cdot 6}{6^2}\right)^3 = 4^3 6^3 = 4^3 6 = 384
\]

2. \[
4^{2 \cdot 3} = 15^{2 \cdot 3^{-4}} = \frac{4^2 15^2}{8^3 3^4} = \frac{4^2 (3 \cdot 5)^2}{(2 \cdot 4)^3 3^4} = \frac{4^2 2^2 5^2}{2^3 3^2 4^3} = \frac{5^2}{2^3 3^2 4^3} = 25
\]

3. \[
(2^4 2^{-3})^3 = (2^4 2^{-3})^3 = 2^5 = 32
\]

4. \[
a^{\frac{3}{2}} = (a^3)^{\frac{1}{2}} = (16)^{\frac{1}{2}} = 4
\]

5. \[
a = (a^3)^{\frac{1}{3}} = (27)^{\frac{1}{3}} = 3
\]

6. \[
(ab)^8 = (a^4 b^4)^2 = (a^2 b^2)^4 = ((5)^2 81)^2 = 5^4 81^2 = 4, 100, 625. \text{ One could also realize that } a = 5^{\frac{1}{2}} \text{ and } b = 81^{\frac{1}{4}}. \text{ Thus, } (ab)^8 = (5^{\frac{1}{2}} 81^{\frac{1}{4}})^8 = 5^4 81^2.
\]

7. \[
(a^2 b)^3 = (a^3)^2 (b^{-3})^{-1} = 7^2 (5)^{-1} = \frac{7^2}{5} = \frac{49}{5}
\]

8. 

9. 

10. 

11. Since \(4 < 4.5\), the graph of \(4^x\) will lie below the graph of \(4.5^x\) for \(x > 0\) and above it for \(x < 0\).

12. \(2^x\) gets bigger faster than \(x^2\) as is shown in the plot below.
13.

\[
\begin{array}{cccccc}
\text{x} & 2 & 4 & 8 & 16 \\
2^x & 4.0 & 16.0 & 256.0 & 65,536.0 \\
3^x & 8.0 & 64.0 & 512.0 & 4,096.0
\end{array}
\]

From this table it appears that \(2^x\) gets bigger faster than \(x^3\) even though \(x^3\) is bigger for smaller values of \(x\).

\[
\begin{array}{ccccccc}
\text{x} & 2 & 4 & 8 & 16 & 32 & 64 \\
2^x & 4.0 & 16.0 & 256.0 & 65,536.0 & 4.294\times10^9 & 1.844\times10^{19} \\
x^{10} & 1024.0 & 1,048,576.0 & 1.073\times10^9 & 1.0995116\times10^{12} & 1.1258999\times10^{15} & 1.1529215\times10^{18}
\end{array}
\]

From this table it appears that \(2^x\) gets bigger faster than \(x^{10}\) even though \(x^{10}\) is bigger for smaller values of \(x\).

16. a. \(4^x\), b. \(3^{-x}\), and c. \(2^{-x}\)

17. Since the graph passes through the point \((0, 5)\) we must have \(k = 5\). The other data point \((2, 45)\) then gives us the equation

\[
45 = 5a^2 \implies 9 = a^2 \implies 3 = a.
\]

18. \(e^{x+y} = e^x e^y = 3 \cdot 7 = 21\)

19. From the formula \(e^x = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n\), we have \(e^3 = \lim_{n \to \infty} \left(1 + \frac{3}{n}\right)^n\). Evaluating the expression \(\left(1 + \frac{3}{n}\right)^n\) at \(n = 50,000\), we have

\[
e^3 \approx 20.083729
\]

Using our calculator we have

\[
e^3 = 20.085537.
\]

20. \(e^{2t} = (e^t)^2 = 5^2 = 25\)

21. \(e^5\) must satisfy the inequality

\[
2^5 < e^5 < 3^5 \text{ or } 32 < e^5 < 243
\]

22.

\[
e^5 - 2.71828^5 = 148.413159102577 - 148.412659950842 = 0.000499151735
\]

23. From \(2.7 < e < 2.8\) we have

\[
\frac{1}{2.8} < \frac{1}{e} < \frac{1}{2.7}
\]

\[
.3571428 < \frac{1}{e} < .3703703
\]

Remember too, that \(\frac{1}{e} = e^{-1}\).