Maximum/Minimum Functional Values

Local Maximum and Minimum Values

Observe the graph below. If we were walking along the graph, moving from left to right, we would be traveling downhill at the beginning. We would reach a low-point and begin travelling uphill until we reached a high point from which we would travel downhill again. After the next low point we continue uphill for the rest of the way.

Critical points occur where the graph turns around. These are not necessarily the lowest or highest functional values for the entire graph, but they are high points or low points for a particular area of the graph. These points are called generically local extreme points, and may either be local maximum or local minimum points.

The graph changes from decreasing to increasing at local minimum points.

The graph changes from increasing to decreasing at local maximum points.

It takes the methods of calculus to determine these points without a calculator; however, we can approximate these values using a graphing calculator.

Example 1: Use a graphing calculator to find the local extrema rounded to the nearest hundredth for the function \( f(x) = x^3 - 8x + 1 \). Use the extreme points to determine the intervals where \( f \) is increasing and decreasing.

Solution:

- local maximum functional value of 9.71 when \( x \approx -1.63 \)
- local minimum functional value of -7.71 when \( x \approx 1.63 \)
- Increasing: \((−\infty, -1.63) \cup (1.63, +\infty)\)
- Decreasing: \((-1.63, 1.63)\)