Maximum/Minimum Functional Values

Local Extreme Values: Applications

Example 1: An open box is constructed from a 12-inch square piece of cardboard by cutting squares of equal length from each corner and turning up the sides. What should the dimensions of the cutout square be for the box to have a maximum volume?

Solution: Express volume as a function of the length of the cutout square:

- Let \( x \) = number of inches on one side of the cutout square
- \( V \) = number of cubic inches in volume of the box
  Then \( V(x) = x(12 - 2x)^2 \)

- The domain must be restricted to \([0, 6]\) because the value of \( x \) represents a length and must be non-negative. Moreover, since the side of the piece of cardboard is 12 inches and we can cut no more than half of 12, we must also have \( x \leq 6 \).
- Enter the function \( x(12 - 2x)^2 \) into a graphing calculator over the restricted domain \([0, 6]\).

The maximum volume of the box is 128 \( \text{in}^3 \) if a 2-inch square is cut out of each corner of the cardboard.

Question: Why couldn’t we analyze this problem as we did the previous problems?

Answer: The function whose extreme value we wanted to find is a cubic polynomial. At this time we only know how to analyze quadratic polynomials. You will learn how to analyze cubics and even more complicated functions if you study calculus.