Applying Functions

Algebra is a tool that can help us in many circumstances in our personal lives as well as our careers. Any time you enter a formula into a spreadsheet you are using algebra. In fact, many of the times you use algebraic thinking you are not aware that you are using it. In either instance, the ability to formulate quantitative problems mathematically is critical to use algebra as a problem solving tool. In this section we will begin the process by translating some situations into functions.

Example 1: The Student Government is considering ways to raise funds for a gift they wish to purchase for the school. One suggestion is to sponsor a Back to School Dance at the Brazos Center at the beginning of the semester. Rent for the Brazos Center is $150, and the social chairman knows a deejay who will only charge $100 for the event. A local caterer has agreed to provide the refreshments for $2/ticket sold. Suppose the SG charges $5 per ticket.

a) How many tickets must be sold to break even?

b) How much will they clear if they sell a ticket to 10% of the 7600 students enrolled at Blinn-Bryan?

c) How many tickets must be sold if they are to fully fund the gift which costs $300?

d) Use your grapher to draw a plot of the function. Label the axes and include the number scale for each axis.

e) If you were on the executive committee of SG, how would you vote? Why?

Solution: We will write a function to model the information we have: Let \( x \) = the number of tickets sold. Since the proceeds depend on the number of tickets sold, \( p(x) = \) the proceeds if \( x \) tickets are sold. Since proceeds = revenue - costs,

\[
p(x) = 5x - (2x + 250) = 3x - 250
\]

a) To break even, \( p(x) = 0 \) \( \Rightarrow 3x - 250 = 0 \) \( \Rightarrow 3x = 250 \) \( \Rightarrow x = \frac{250}{3} = 83 \frac{1}{3} \) \( \Rightarrow \)

84 tickets must be sold.

b) If 10% of 7600 = 760 students buy tickets, \( x = 760 \) \( \Rightarrow p(760) = 3(760) - 250 = 2030 \)
Proceeds will be $2030 if 10% of students buy tickets.

c) For proceeds of $300, \( 300 = 3x - 250 \) \( \Rightarrow 3x = 550 \) \( \Rightarrow x = \frac{550}{3} = 183 \frac{1}{3} \) \( \Rightarrow \) 184 tickets must be sold to buy gift.

d) [Graph of the function showing the relationship between tickets sold and total proceeds.]

e) Your opinion here...
Example 2: A. J. Ramirez, Inc. purchased a strip shopping center for $1.5 million. If the depreciation is $50,000 per year for 30 years, find the following:

a) Write a function for the current value of the shopping center.
b) Determine the domain.
c) Use your grapher to draw a plot of the function. Label the axes and include the $x$-scale and $y$-scale.
d) Find the value of the center in 20 years.
e) When will the center be worth $1 million?

Solution:

a) Let $x$ = the number of years since the shopping center was purchased.

$v$ = the current value of the center.

Then $v(x) = 1500000 - 50000x$.

b) We must deal with a non-negative number of years, so $x \geq 0$. But the building is completely depreciated in 30 years. Therefore, the domain is $[0, 30]$.

c) 

<table>
<thead>
<tr>
<th>Value at $x$ years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500000</td>
</tr>
<tr>
<td>1000000</td>
</tr>
<tr>
<td>500000</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

d) $v(20) = 1500000 - 50000(20) = 500000 \Rightarrow$ The building is worth $500,000 in 20 years.

e) We are given $v(x) = 1,000,000$ and we must find the number of years $x$.

Solve for $x$: $1500000 - 50000x = 1000000 \Rightarrow -50000x = -500000 \Rightarrow x = 10$

The building will be worth $1 million in 10 years. Note that the graph supports the algebraic solution.

Remember the key to determining if an equation represents a function is whether there is one output for each input in the domain of the function. For example, equations of circles do not represent functions, but most lines do represent functions.

**Question:** What kind of line does not represent a function?

**Answer:** Vertical line.

In this section we’ll look at some examples of geometric formulas which are functions.

Consider the formula for the area of a square $A = s^2$. The input is the length of the side of the square $s$ and the output is the area of that particular square. Using function notation, we would write $A(s) = s^2$. Although the domain of function $A$ is normally $(-\infty, +\infty)$, in this case we would restrict the domain to $[0, +\infty)$ because the function is an application of a geometric formula. The lengths of sides and areas of squares, as well as other geometric figures, are always non-negative.

**Example 3:** Write the perimeter of a square (the distance around the figure) in terms of the length of one of its sides. Find the domain of the resulting function.

**Solution:**

$p(s) = 4s \quad $Domain: $[0, +\infty)$
Example 4: Use the above formulas involving squares to complete the table below:

<table>
<thead>
<tr>
<th>$p$</th>
<th>20</th>
<th>64</th>
<th>100</th>
<th>84</th>
<th>68</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution: You know that $p = 4s$, so you probably divided the perimeter by 4 to get the length of one side of the square. To find the area, you squared the length of the side, because $A = s^2$.

Example 5: We can even write a function to describe the process you used to complete the above table. That is, write $A$ as a function of $p$.

Solution: Since $A$ and $p$ are both functions of $s$, we can eliminate the $s$ by first solving $p = 4s$ for $s$:

$s = \frac{P}{4}$. Substituting $\frac{P}{4}$ for $s$ into $A = s^2$, we get $A = \left(\frac{P}{4}\right)^2$. In functional notation we write $A(p) = \frac{P^2}{16}$.

Check this function out for each input in the table above. For example, $A(20) = \frac{(20)^2}{16} = 25$. Isn’t this the result you obtained in the above example?

Example 6: Write the perimeter of a square $p$ as a function of $A$. [$p(A)$]

Solution: That means, we want $p$ to be the output and $A$ to be the input. Start with the function you have for the output.

- Start with the function for $p$: $p = 4s$
- Solve $A = s^2$ for $s$: $s = \sqrt{A}$
- Replace the input $s$ with its equivalent $\sqrt{A}$ in to $p = 4s$: $p = 4\left(\sqrt{A}\right) \Rightarrow p(A) = 4\sqrt{A}$.

Example 7: Use the formulas for the circumference ($C = 2\pi r$) and area ($A = \pi r^2$) of a circle to write each of the following functions. Restrict their domain as necessary.

- a) write the radius as a function of the circumference. [$r(C)$]
- b) write the radius as a function of the area. [$r(A)$]
- c) write the circumference as a function of the area. [$C(A)$]
- d) write the area as a function of the circumference. [$A(C)$]

Solution:

a) Solve $C = 2\pi r$ for $r$: $r = \frac{C}{2\pi}$ \Rightarrow in function notation: $r(C) = \frac{C}{2\pi}$ Domain: $[0, +\infty)$

b) Solve $A = \pi r^2$ for $r$: $r^2 = \frac{A}{\pi} \Rightarrow r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{A \cdot \pi}{\pi \cdot \pi}} = \frac{\sqrt{A\pi}}{\pi}$ (We only want the positive root.)

Domain: $[0, +\infty)$

c) Start with $C = 2\pi r$. Solve $A = \pi r^2$ for $r$: $r = \frac{\sqrt{A\pi}}{\pi}$ (See results of b)

Substitute for $r$: $C = 2\pi \left(\frac{\sqrt{A\pi}}{\pi}\right) = 2\sqrt{A\pi} \Rightarrow C(A) = 2\sqrt{A\pi}$ Domain: $[0, +\infty)$

d) Start with $A = \pi r^2$. Solve $C = 2\pi r$ for $r$: $r = \frac{C}{2\pi}$ Substitute for $r$:

$A = \pi \left(\frac{C}{2\pi}\right)^2 = \pi \left(\frac{C^2}{4\pi^2}\right) = \frac{C^2}{4\pi} \Rightarrow A(C) = \frac{C^2}{4\pi}$ Domain: $[0, +\infty)$
Suppose we wanted to write the area of a rectangle as a function of the length of one of its sides.

Let \( x \) = the number of units in the length of one side of the rectangle
and \( y \) = the number of units in the length of the other side of the rectangle
and \( A \) = the number of square units in the area of the rectangle.

We know that the relationship between the area and the length and width of the rectangle is \( A = xy \) and the perimeter is \( p = 2x + 2y \).

We start with the formula for the area of the rectangle:

\[
A = xy
\]

We can then use the perimeter to write \( y \) in terms of \( x \):

\[
p = 2x + 2y \Rightarrow y = \frac{p - 2x}{2} = \frac{p}{2} - x
\]

Substitute into \( A = xy \): \( A = x\left(\frac{p}{2} - x\right) \Rightarrow A(x) = \frac{p}{2}x - x^2 \)

In the following, we chose \( p = 200 \) so that \( y = \frac{200}{2} - x = 100 - x \) and \( A = x(100 - x) \) or \( A = 100x - x^2 \).

Consider \( A \) in factored form: \( A(x) = x(100 - x) \). Fill in the table below:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>10</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>75</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 100 - x )</td>
<td>100</td>
<td>90</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>25</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>( A(x) )</td>
<td>0</td>
<td>900</td>
<td>2400</td>
<td>2500</td>
<td>2400</td>
<td>1875</td>
<td>900</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that with a limited perimeter, if \( x \) is increasing, then \( y \) must be decreasing. The area will be 0 when \( x = 0 \) or \( y = 0 \).

**Question:** What is the domain of \( A(x) = 100x - x^2 \)?

**Answer:** Since \( x \) represents the length of a side of a rectangle, \( x \) cannot be a negative number, so \( x \geq 0 \).

But if \( x > 100 \), then \( y \) will be a negative number. Therefore, the domain is \([0, 100]\).

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**Example 8:** A farmer wishes to fence a rectangular area adjacent to a straight brick wall so he doesn’t need to fence along the wall. He also wants to divide the fenced area into two smaller areas as shown below. If he has a total of 4000 feet of fencing material, write a function for the total fenced area in terms of the length of one of the sides.

Solution: Let \( x \) = the number of feet of one side of the rectangle, \( y \) = the number of feet of the other side, and \( A \) = the number of square feet in the area. Then \( A = xy \). Since 4000 feet of fencing \( \Rightarrow 4000 = y + 3x \Rightarrow y = 4000 - 3x \). We can substitute \( y = 4000 - 3x \) for \( y \): \( A = x(4000 - 3x) \Rightarrow A(x) = 4000x - 3x^2 \).

**Question:** What is the domain of \( A(x) = x(4000 - 3x) \)?

**Solution:** If \( x = 0 \), \( A(0) = 0 \). If \( y = 0 \), then \( 4000 - 3x = 0 \Rightarrow x = \frac{4000}{3} = 1333\frac{1}{3} \)

Domain: \([0, 1333\frac{1}{3}]\)