Graphs of Functions

Domain and Range from Graphs

We can find the domain and range of a function by reading its graph. The domain is the reflection of the graph onto the x-axis, while the range is the reflection of the graph onto the y-axis as shown below.

\[(a, f(a))\]

\[\downarrow\]

\[\text{RANGE}\]

\[y = f(x)\]

\[\leftarrow (b, f(b))\]

\[\text{DOMAIN}\]

Domain: \([a, b]\)  Range: \([f(b), f(a)]\)

Example 1: Find the domain and range of \(f(x) = \sqrt{x + 3}\).

Solution: The graph of \(f\) is

To find the domain of \(f\) imagine a vertical line scanning the x-axis from \(-\infty\) to \(+\infty\). If the line intersects the graph at point \((x, y)\), \(x \in \text{Domain}_f\).

To find the range of \(f\) imagine a horizontal line scanning the y-axis from \(-\infty\) to \(+\infty\). If the line intersects the graph at point \((x, y)\), \(y \in \text{Range}_f\).

\[\text{Domain}_f = [-3, +\infty)\]

Vertical lines intersect the graph when \(x \geq -3\).

\[\text{Range}_f = [0, +\infty)\]

Horizontal lines intersect the graph when \(y \geq 0\).
Example 2: Use the graphs to find the domain and range of each function:

a. 

b. 

c. 

Solution:

a. Domain: \([-3, 3]\]  
   Range: \([0, 3]\]

b. Domain: \((-\infty, +\infty)\]  
   Range: \((-\infty, +\infty)\]

c. Domain: \((-\infty, +\infty)\]  
   Range: \((-\infty, 4]\)