Piecewise Functions

Sometimes one individual rule does not describe the function. For example, one company’s cell phone plan charges $29/month for 90 minutes and an additional $0.35/minute for every minute over 90 minutes. The function \( c(x) \) describing the monthly costs of operating the cell phone for \( x \) minutes would be

\[
c(x) = \begin{cases} 
29 & 0 \leq x \leq 90 \\
29 + 0.35(x - 90) & x > 90
\end{cases}
\]

Functions of this nature are called piecewise functions because they are defined in pieces.

Example 1: Graph \( f(x) = \begin{cases} 
x, & x < 0 \\
x^2, & x \geq 0
\end{cases} \) by plotting points.

Solution: If \( x < 0 \), find \( f(x) \) by substituting into \( f(x) = x \).
If \( x \geq 0 \), find \( f(x) \) by substituting into \( f(x) = x^2 \).
Example 2: Graph the piecewise function: \( f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \).

Solution:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
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</thead>
<tbody>
<tr>
<td>-3</td>
<td>3</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
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Question: Which basic function is graphed above? How do you know they are the same function?

Answer: The absolute values function \( f(x) = |x| \). The algebraic definition of absolute value is \( |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \). Therefore, \( f(x) = |x| \) and \( f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \) are the same functions.

Note further that \( f \) is the part of the line \( y = x \) when \( x \geq 0 \) and part of the line \( y = -x \) when \( x < 0 \).

Example 3: Graph the piecewise function: \( f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ x + 2 & \text{if } x < 0 \end{cases} \).

Solution: The graph of \( f \) consists of the part of \( f(x) = x^2 \) where \( x \geq 0 \), and the line \( y = x + 2 \) where \( x < 0 \) as shown below.

To show that the point \((0,2)\) is not included in the graph, we use an open circle. To show that the point \((0,0)\) is included, we color in the point. It is important to indicate clearly that one of the points when \( x = 0 \) is NOT included. If both are included the graph does not pass the vertical line test at \( x = 0 \).
Example 4: Graph the piecewise function: \( f(x) = \begin{cases} x^2 & \text{if } x \geq 1 \\ 3 & \text{if } x = 1 \\ -x + 1 & \text{if } -1 \leq x < 1 \end{cases} \).

Solution:

Questions:

a) What is the domain of \( f \)?
b) What is the range?
c) Evaluate \( f(1) \).
d) What are the intercepts?

Answers:

a) Domain: \([-1, +\infty)\)
b) Range: \([0, +\infty)\)
c) \(f(1) = 3\).
d) y-intercept: 1 \( x \)-intercept: \(-1\)