Matrices

Applications

Besides appearing in systems of equations matrices are often used to model various events. In the next few pages we give several examples of this.

Example 1: A manufacturer of women’s apparel makes two sizes of gloves: medium and small. Suppose it takes 2 square feet of leather to make 1 pair of small gloves and 2.5 square feet to make one pair of medium size gloves. Suppose it also takes 15 minutes from start to finish and 20 minutes from start to finish to make a small and a medium pair respectively. How much leather and time will it take to make 5 pairs of small and 7 pairs of medium size gloves?

Solution: Since each pair of small gloves takes 2 square feet, 5 pair will require 2 \( \times \) 5 square feet; similarly it will require 2.5 \( \times \) 7 square feet to make 7 pair of medium sized gloves. Thus, the total amount of leather is

\[
\text{amount of leather} = 2 \times 5 + 2.5 \times 7 = 27.5
\]

To determine the amount of time needed we perform similar calculations. It will require 15 \( \times \) 5 minutes to make 5 pairs of small gloves and 20 \( \times \) 7 minutes to make 7 pair of medium gloves. Thus, the total amount of time it will take is:

\[
\text{time in minutes} = 15 \times 5 + 20 \times 7 = 215
\]

We next write these equations in a matrix form.

\[
\begin{bmatrix}
\text{amount of leather} \\
\text{time in minutes}
\end{bmatrix} =
\begin{bmatrix}
2 \times 5 + 2.5 \times 7 \\
15 \times 5 + 20 \times 7
\end{bmatrix} =
\begin{bmatrix}
2 & 2.5 \\
15 & 20
\end{bmatrix}
\begin{bmatrix}
5 \\
7
\end{bmatrix}
\]

The entries of the above 2 \( \times \) 2 matrix are the per unit costs, in leather and time, for each type of glove.

This matrix can be used to compute the total amount of material and time needed for every number of small and medium sized gloves. In this particular example we wanted to make 5 small and 7 medium. To find the totals of the materials needed we merely multiply the above matrix and

\[
\begin{bmatrix}
5 \\
7
\end{bmatrix}
\]

The result of this multiplication gives us the required totals:

\[
\begin{bmatrix}
2 & 2.5 \\
15 & 20
\end{bmatrix}
\begin{bmatrix}
5 \\
7
\end{bmatrix} =
\begin{bmatrix}
27.5 \\
215
\end{bmatrix}
\]

Thus, we require 27.5 square feet of leather and 215 minutes.
Question: How much leather and time is needed to produce 15 and 30 small and medium pairs of gloves respectively?
Answer: 105 square feet of leather and 825 minutes.

$$\begin{bmatrix} 2 & 2.5 \\ 15 & 20 \end{bmatrix} \begin{bmatrix} 15 \\ 30 \end{bmatrix} = \begin{bmatrix} 105.0 \\ 825 \end{bmatrix}$$

Example 2: Concrete is basically a mixture of gravel and cement. The size of the gravel and the amount of cement determine the strength of the concrete. For this example, we assume that gravel comes in two sizes: fine which means no particles larger than 3/16 of an inch in diameter, and coarse which means particle sizes range from 3/16 to 3/4 of an inch in diameter. The company "Never Crack" sells three grades of concrete: standard grade, home grade, and commercial grade, each one being stronger than the preceding one. The table below shows how much of each type of gravel and cement is used to produce one yard of each of these three grades of concrete. Suppose Never Crack receives an order for 50 yards of standard concrete, 100 yards of home concrete and 250 yards of commercial concrete. How much of each type of gravel and cement will be required to fill the order?

<table>
<thead>
<tr>
<th></th>
<th>fine</th>
<th>coarse</th>
<th>cement</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard</td>
<td>1/2 yard</td>
<td>1/2 yard</td>
<td>3.5 bags</td>
</tr>
<tr>
<td>home</td>
<td>1/3 yard</td>
<td>2/3 yard</td>
<td>4.5 bags</td>
</tr>
<tr>
<td>commercial</td>
<td>1/4 yard</td>
<td>3/4 yard</td>
<td>6 bags</td>
</tr>
</tbody>
</table>

Solution: We first construct the matrix whose entries are the per unit amounts of material needed for each type of material used. To ensure we place the entries in the matrix correctly we construct a model equation. Suppose we want $x$ yards of standard cement, $y$ yards of home grade and $z$ yards of commercial. Then we will require the following amount of fine gravel

$$\frac{1}{2}x + \frac{1}{3}y + \frac{1}{4}z$$

This expression prompts us to construct the following matrices

$$\begin{bmatrix} 1/2 & 1/3 & 1/4 \\ 1/2 & 2/3 & 3/4 \\ 3.5 & 4.5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The solution to our problem is given by the matrix product

$$\begin{bmatrix} 1/2 & 1/3 & 1/4 \\ 1/2 & 2/3 & 3/4 \\ 3.5 & 4.5 & 6 \end{bmatrix} \begin{bmatrix} 50 \\ 100 \\ 250 \end{bmatrix} = \begin{bmatrix} 725/6 \\ 1675/6 \\ 2125.0 \end{bmatrix}$$

Thus, Never Crack needs 725/6 yards of fine gravel, 1675 yards of coarse gravel, and 2125 bags of cement to fill the order.

Question: Suppose Never Crack decides to change the amount of material in the commercial grade to the following proportions 1/8 yard of fine gravel, 7/8 yard of coarse gravel, and 5.75 bags of cement. What would be changed in the matrix we constructed?

Answer: Column 3 would change. The new entries of column 3 are 1/8, 7/8, and 5.75 from top to bottom.