Matrices

Exercises

1. What does $a_{14}$ equal if $A = \begin{bmatrix} -3 & 1 & 0 & 11 \\ 27 & 7 & 13 & 5 \end{bmatrix}$?

2. What does $a_{34}$ equal if $A = \begin{bmatrix} -3 & 1 & 0 & 11 \\ 27 & 7 & 13 & 5 \end{bmatrix}$?

3. If $A$ is a $2 \times 2$ matrix with $a_{11} = 0, a_{12} = -5, a_{21} = 1,$ and $a_{22} = 6$, what does $A$ equal?

4. $21 \begin{bmatrix} -4 & 5 \\ 3 & 2 \end{bmatrix}$ equals?

5. $\begin{bmatrix} -4 & 5 \\ 3 & 2 \end{bmatrix} - 3 \begin{bmatrix} 1 & -11 \\ -1 & 5 \end{bmatrix}$ equals?

6. Can you add the following two matrices to each other? $\begin{bmatrix} -1 & 12 \\ 7 & -4 \end{bmatrix}$, $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. If yes, add them, if no, why not?

7. What is the size of $AB$ if $A$ is a $2 \times 3$ matrix and $B$ is a $3 \times 7$ matrix?

8. If $A$ is a $5 \times 6$ matrix and $B$ is a $4 \times 5$ matrix, which, if either, of the products $AB$ or $BA$ is defined. If the product is not defined explain why, and if the product is defined what is the size of the product matrix?

9. $2 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + 5 \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix}$ equals?

10. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \end{bmatrix}$ equals?

11. $\begin{bmatrix} -1 & 12 \\ 7 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ equals?

12. $\begin{bmatrix} 3 & 22 \\ \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ equals?

13. Suppose that $\begin{bmatrix} 2 & x \\ \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 9$, what must $x$ equal?
14. Does there exist an \( x \) such that \[
\begin{bmatrix}
2 & x \\
3 & -2
\end{bmatrix}
\begin{bmatrix}
1 & 7 \\
2 & 0
\end{bmatrix}
= 
\begin{bmatrix}
2 & 10 \\
5 & 0
\end{bmatrix}
\]?

15. Does there exist an \( x \) such that \[
\begin{bmatrix}
1 & x \\
3 & -2
\end{bmatrix}
\begin{bmatrix}
-5 & 9 \\
0 & x
\end{bmatrix}
= 
\begin{bmatrix}
-5 & 10 \\
-15 & 29
\end{bmatrix}
\]?

16. Suppose that \[
\begin{bmatrix}
11 & x \\
0 & -4
\end{bmatrix}
+ 2
\begin{bmatrix}
y & 6 \\
2 & -7
\end{bmatrix}
= 
\begin{bmatrix}
-1 & 12 \\
4 & -26
\end{bmatrix}
\], what must \( x \) and \( y \) equal?

17. Let \( A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 5 \\ -5 & 7 & 0 \end{bmatrix} \) and \( B = \begin{bmatrix} 2 & 0 & 12 \\ -8 & 1 & 1 \\ 13 & 2 & 9 \end{bmatrix} \).
   a. \( 2A - B =? \)
   b. \( AB =? \)
   c. \( BA =? \)

18. What is the coefficient matrix for the system of equations below?
\[
2x_1 + 7x_2 = 3
\]

19. What is the augmented matrix for the system \( 2x_1 + 7x_2 = 3 \)?

20. What are the coefficient matrix and augmented matrix for the system below
\[
\begin{align*}
3x_1 - 5x_2 &= 8 \\
x_1 + 7x_2 &= 9
\end{align*}
\]

21. If \[
\begin{bmatrix}
4 & 3 & -2 \\
2 & 1 & 31
\end{bmatrix}
\] is the augmented matrix of a system of equations, what is the system?

22. What is the inverse of the matrix \[
\begin{bmatrix}
4 & 3 \\
2 & 1
\end{bmatrix}
\]?

23. What is the inverse of \[
\begin{bmatrix}
4 & 3 \\
0 & 1
\end{bmatrix}
\]?

24. If \[
\begin{bmatrix}
2 & 5 \\
3 & 7
\end{bmatrix}
\] is the inverse of \( A \), what must \( A \) equal?

25. Solve the system of equations below by writing the system as a matrix equation, and then multiply the matrix equation by the inverse of the coefficient matrix
\[
\begin{align*}
2x - 3y &= 7 \\
2x + 3y &= 5
\end{align*}
\]
26. If \( A \) is a 3 \( \times \) 5 matrix and \( B \) is a 3 \( \times \) 4 matrix, what size must \( X \) be if \( X \) is a solution to the equation \( AX = B \)?

27. If \( A \) is a 3 \( \times \) 5 matrix and \( B \) is a 3 \( \times \) 5 matrix, what size must \( X \) be if \( X \) is a solution to the equation \( XA = B \)?

28. Solve the matrix equation \( AX = B \) where \( A = \begin{bmatrix} -3 & 1 \\ 1 & 2 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \).

29. The determinant of \( \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \) equals?

30. Compute the determinant of each of the following matrices.
   a. \( \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \)
   b. \( \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \)
   c. \( \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \)

31. Show that for any \( a \), \( b \), and \( k \) the determinant of the matrix \( \begin{bmatrix} a & b \\ ka & kb \end{bmatrix} \) must equal zero.

32. Show that for any \( k \) the determinant of the matrix \( \begin{bmatrix} a & b & c \\ ka & kb & kc \\ d & e & f \end{bmatrix} \) must equal zero.

33. How are the determinants of the matrices \( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) and \( \begin{bmatrix} c & d \\ a & b \end{bmatrix} \) related?

34. Compute the inverse of the matrix \( \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \).

35. Compute the inverse of \( \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \).
36. Let \( A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \) and \( B = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \). Compute \( AB \) and \( BA \).

37. Solve the system below by converting it to a matrix equation and then multiplying by the inverse of the coefficient matrix.

\[
\begin{align*}
2x_1 - x_2 + x_3 &= 1 \\
x_1 + x_2 + x_3 &= 0 \\
x_1 + 2x_2 + 3x_3 &= 4
\end{align*}
\]

38. Use Cramer’s rule to solve the system:

\[
\begin{align*}
2x - 5y &= 3 \\
x + 7y &= -8
\end{align*}
\]

39. Solve the system below using Cramer’s rule.

\[
\begin{align*}
x + y + z &= 1 \\
x - y + z &= -1 \\
x + 2y - 3z &= 0
\end{align*}
\]

40. Use Cramer’s rule to find \( x \).

\[
\begin{align*}
12x - 5y &= 9 \\
-3x + 7y &= 17
\end{align*}
\]

41. Use Cramer’s rule to find \( y \).

\[
\begin{align*}
x - y + 2z &= 13 \\
2y + 19z &= 11 \\
11x - 5z &= 3
\end{align*}
\]

42. Concrete is basically a mixture of gravel and cement. The size of the gravel and the amount of cement determine the strength of the concrete. For this example, we assume that gravel comes in two sizes: fine which means no particles larger than 3/16 of an inch in diameter, and coarse which means particle sizes range from 3/16 to 3/4 of an inch in diameter. The company “Never Crack” sells three grades of concrete: standard grade, home grade, and commercial grade, each one being stronger than the preceding one. The table below shows how much of each type of gravel and cement is used to produce one yard of each of these three grades of concrete. Suppose Never Crack receives an order for 200 yards of standard concrete and 450 yards of commercial concrete. How much of each type of gravel and cement will be required to fill the order?

<table>
<thead>
<tr>
<th></th>
<th>fine</th>
<th>coarse</th>
<th>cement</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard</td>
<td>1/2 yard</td>
<td>1/2 yard</td>
<td>3.5 bags</td>
</tr>
<tr>
<td>home</td>
<td>1/3 yard</td>
<td>2/3 yard</td>
<td>4.5 bags</td>
</tr>
<tr>
<td>commercial</td>
<td>1/4 yard</td>
<td>3/4 yard</td>
<td>6 bags</td>
</tr>
</tbody>
</table>

43. In the above example, Never Crack, decides to change the amounts of material in all three of its concrete products to the following.

<table>
<thead>
<tr>
<th></th>
<th>fine</th>
<th>coarse</th>
<th>cement</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard</td>
<td>3/7 yard</td>
<td>4/7 yard</td>
<td>3 bags</td>
</tr>
<tr>
<td>home</td>
<td>1/3 yard</td>
<td>2/3 yard</td>
<td>4 bags</td>
</tr>
<tr>
<td>commercial</td>
<td>1/9 yard</td>
<td>8/9 yard</td>
<td>5.75 bags</td>
</tr>
</tbody>
</table>

How much of each material will Never Crack need to an order of 100 yards of standard, 150 yards of home grade, and 50 yards of commercial grade concrete?