Polynomials

Definition of a Polynomial

A polynomial is the most common algebraic expression. Polynomials are expressions which contain terms of the form $ax^k$, where $a$ is a real constant and $k$ is a nonnegative integer. More formally, a polynomial expression is an expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

In the above expression, $a_n$ is a nonzero real number, $a_0, a_1, \ldots, a_{n-1}$ are real numbers, and $n$ is a positive integer. $n$ is called the degree of the polynomial, $a_n$ is called the leading coefficient, and $a_0$ is called the constant term.

If $n = 2$ the polynomial is called a binomial or quadratic.
If $n = 3$ the polynomial is called a trinomial or cubic.

Example The height, $s$ (in feet), of a ball thrown in the air at time $t = 0$ seconds is given by the polynomial expression $s = -16t^2 + 80t + 5$. Here the degree of the polynomial is 2 (the highest exponent), the leading coefficient is $-16$, and the constant is 5.

Example The volume, $V$, of a cube with edge length $x$ is given by $V = x^3$. Here the degree of the polynomial is 3, the leading coefficient is 1 (remember that $x^3 = 1x^3$), and the constant is 0.

Polynomial expressions do not contain negative exponents nor do they contain radicals. So an expression such as $\sqrt{9 - x^2}$ is not a polynomial.

Question: Is $x^3 + \frac{1}{x}$ a polynomial?
Answer: No, the $\frac{1}{x}$ term is the same as $x^{-1}$ and the exponent is not a positive integer.