Exponents and Radicals

Combining Radical Expressions

Radical expressions can only be combined if there are like terms—terms with the same index and same radicand (the expression inside the radical).

\[ \sqrt{2} + 2\sqrt{3} + 4\sqrt{2} - 4\sqrt{3} \]

In the above expression, the terms with \( \sqrt{2} \) are alike and the terms with \( \sqrt{3} \) are alike. Therefore, the above expressions simplifies as follows:

\[
\begin{align*}
\sqrt{2} + 2\sqrt{3} + 4\sqrt{2} - 4\sqrt{3} &= \sqrt{2} + 4\sqrt{2} + 2\sqrt{3} - 4\sqrt{3} \\
&= (1 + 4) \sqrt{2} + (2 - 4) \sqrt{3} \\
&= 5\sqrt{2} - 2\sqrt{3}
\end{align*}
\]

Notice that the two remaining terms are not alike and, hence, cannot be simplified.

Some terms which do not look alike at first glance may be alike after simplifying. Therefore, it is important that you simplify all radicals before combining like terms.

Examples:

- \( \sqrt{50} - \sqrt{32} + \sqrt{2} = \sqrt{25 \cdot 2} - \sqrt{16 \cdot 2} + \sqrt{2} \\
  = 5\sqrt{2} - 4\sqrt{2} + \sqrt{2} \\
  = (5 - 4 + 1) \sqrt{2} \\
  = 2\sqrt{2} \)

- \( 7\sqrt{80x} - 2\sqrt{270x} + 4\sqrt{10} = 7\sqrt{4 \cdot 5x} - 2\sqrt{9 \cdot 30x} + 4\sqrt{10} \\
  = (7 \cdot 2) \sqrt{2 \cdot 5x} - (2 \cdot 3) \sqrt{2 \cdot 30x} + 4\sqrt{10} \\
  = 14\sqrt{10x} - 6\sqrt{10x} + 4\sqrt{10} \\
  = 8\sqrt{10x} + 4\sqrt{10} \)

  (note that these terms are NOT alike even though they both have a 10)

- In simplifying \( \sqrt{\frac{1}{2}} - \sqrt{\frac{1}{8}} \), we begin by using Property 3 (\( \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \)), then simplifying and rationalizing the denominator before combining:

\[
\begin{align*}
\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{8}} &= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{8}} \quad \text{(Note that 8 = 4 \cdot 2)} \\
&= \frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}} \quad \text{(Multiply both by \( \frac{\sqrt{2}}{\sqrt{2}} \))} \\
&= \frac{1\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} - \frac{1\sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}} \\
&= \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} \quad \text{(now get a common denominator)} \\
&= \frac{2\sqrt{2}}{4} - \frac{\sqrt{2}}{4} \\
&= \frac{2\sqrt{2} - \sqrt{2}}{4} \quad \text{(now combine radicals)} \\
&= \frac{\sqrt{2}}{4}
\end{align*}
\]