Rational Functions

Exercises

1. Which of the following functions is rational?
   a. $\frac{2x}{x^2 + 2x - 5}$
   b. $\frac{x^2 - 1}{x^4 + 2x - 5}$
   c. $\frac{x + x^3 - 3}{2x}$
   d. $x^2 - 3x + 5$

2. Is $f(x) = \frac{x - 1}{\sqrt{x}}$ a rational function.

3. Find the domain of each of the following functions.
   a. $\frac{x - 1}{x^2 - 1}$
   b. $\frac{2x^3 - 5x + 8}{x^5 - 8}$
   c. $\frac{x^5 - 1}{x^2 + x - 2}$

4. Find the range of the following functions.
   a. $\frac{1}{x - 2}$
   b. $\frac{1}{x^2} - 1$
   c. $\frac{3 - 2}{x + 5}$

In problems 5 through 9 find the $x$ and $y$-intercepts of the rational function:

5. $\frac{2x^2 - 6}{x^2 + 1}$
6. $\frac{x^2 - 4}{2x}$
7. $\frac{2x + 1}{x}$
8. $\frac{3}{x^2 - 3x - 4}$
9. $\frac{2x^2 - 3x + 7}{x - 1}$

10. Find the limit as $x \to \infty$ of $\frac{2x - 1}{x}$
11. Find the limit as $x \to \infty$ of $\frac{2x^2 - 1}{x}$
12. Find the horizontal asymptote of $r(x) = \frac{2x^2 - 6}{-x^2 + 2}$ and plot the function
13. Find the limit as $x \to \infty$ of $\frac{x^3 - x^2 + 5x - 18}{19x^4 + 15}$, and plot the function for large values of $x$
14. Does the function $\frac{x^2 - 2x + 7}{x - 1}$ have a horizontal asymptote?
15. Find a rational function which has the line $y = 6$ as a horizontal asymptote.
16. Find a rational function which has 2 as an $x$-intercept and the line $y = -1$ as a horizontal asymptote.
17. Find a rational function which has $-3$ as an $x$-intercept, 7 as a $y$-intercept, and has the line $y = 1$ as a horizontal asymptote.
18. For each of the following functions locate their vertical asymptotes, and then plot the functions.
   a. $\frac{x}{x - 1}$
   b. $\frac{x^2 - 2}{x^2 - 3x + 2}$
   c. $\frac{x - 1}{x^2 + 2x - 3}$

19. Locate all vertical and horizontal asymptotes of $y = \frac{2x^2 - 1}{x^2 - x - 6}$
20. Locate all vertical and horizontal asymptotes of $y = \frac{x^2 - 1}{2x + 3}$
21. Find all intercepts and asymptotes of the function \( y = \frac{x^2 - 4}{-3x^2 + 27} \)
22. Find all asymptotes of the function \( y = \frac{3x^2 - 1}{x^3 - 1} \)
23. Find all asymptotes of the function \( y = \frac{x^4 - x^3 - 4x + 7}{5x^2 - 6} \)
24. Find a rational function which has \( y = 1 \) as a horizontal asymptote and the lines \( x = 3 \) and \( x = 5 \) as vertical asymptotes
25. Find a rational function which has the line \( y = \frac{5}{2} \) as a horizontal asymptote and the lines \( x = 4, x = 5, \) and \( x = -2 \) as vertical asymptotes
26. Which of the following functions have slanted asymptotes?
   a. \( \frac{2x - 6}{x^2 + 9} \)
   b. \( \frac{x^2 - 3x + 1}{5x^2 + 2x + 4} \)
   c. \( \frac{3x^2 + x + 8}{-5x + 2} \)
   d. \( \frac{5x^3 + 1}{2x - 7} \)
27. The rational function \( y = \frac{x^2 - 5x + 1}{2x - 8} \) has a slanted asymptote. What is it?
28. What is the slanted asymptote of the function \( y = x - 6 \)
29. What is the slanted asymptote of the function \( y = \frac{7x^3 - 5x + 8}{3x^2 + 4x - 1} \)
30. Find a rational function which has a vertical asymptote at \( x = 2 \) and the slanted asymptote \( y = -3x + 1 \)

In exercises 31 through 35 determine the domain, \( x \)-intercepts, \( y \)-intercepts, and all asymptotes of the given function.
31. \( \frac{5}{x^2 - 4} \)
32. \( \frac{7 - 2x}{x^2 + 5x + 4} \)
33. \( \frac{3x^2 - 15}{x + 7} \)
34. \( \frac{x^3 - 3x + 5}{-x^2 + 3x - 2} \)
35. \( \frac{x^3 - x - 1}{2x + 1} \)