Rational Expressions

Simplifying Rational Expressions
To simplify a rational expression means to reduce it to lowest terms. From working with fractions, you may recall that simplifying is done by cancelling common factors. Therefore, the key to simplifying rational expressions (and to most problems involving rational expressions) is to factor the polynomials whenever possible.

Example 1: Write \( \frac{8x^4}{12x^6} \) in reduced form.

Solution: Factor the numbers and cancel common factors. Use properties of exponents to help cancel the \( x \)'s:

\[
\frac{8x^4}{12x^6} = \frac{4 \cdot 2 \cdot x^4}{4 \cdot 3 \cdot x^6} = \frac{2}{3x^2}
\]

Example 2: Write \( \frac{y^3 + 5y^2 + 6y}{y^2 - 4} \) in reduced form.

Solution:

\[
\frac{y^3 + 5y^2 + 6y}{y^2 - 4} = \frac{y(y^2 + 5y + 6)}{y^2 - 4} = \frac{y(y + 2)(y + 3)}{(y + 2)(y - 2)} = \frac{y(y + 3)}{y - 2} = \frac{y^2 + 3y}{y - 2}, \quad y \neq \pm 2
\]

Question: Why is \( y \neq \pm 2 \)?
Answer: In order for our answer to be equivalent to the original fraction, the variable must have the same restrictions. Since \( y \) cannot equal \( \pm 2 \) in the original expression (the denominator would then be zero), we must restrict the domain of our answer in order for these fractions to be equivalent.

Example 3: Write \( \frac{4x - x^3}{x^3 - x - 2} \) in reduced form.

Solution:

\[
\frac{4x - x^3}{x^3 - x - 2} = \frac{x(4 - x^2)}{x^3 - x - 2} = \frac{x(2 + x)(2 - x)}{(x - 2)(x + 1)}
\]

At this point, it doesn’t look like anything will cancel. However, if we factor \(-1\) from the last term in the numerator, we obtain the following:

\[
= \frac{-x(2 + x)(x - 2)}{(x - 2)(x + 1)} = \frac{-x^2 - 2x}{x + 1}; \quad x \neq 2 \text{ or } -1
\]

Question: What property of real numbers tells us that \(-x(2 + x) = -x(x + 2)\)?
Answer: The commutative property of addition.

\[ x + 2 = 2 + x. \]