Real Numbers

Properties of Real Numbers
Real numbers are used in almost every human endeavor. Whenever we need to quantify objects, we use numbers. Cooking recipes, prices, interest rates, blood pressure, height, age, voltage, and wind velocity are a few of the everyday objects that are quantified by real numbers. As we know, the two operations of addition (+) and multiplication (\(\times\)) are defined for real numbers. In other words, for any two real numbers \(a\) and \(b\), the sum \(a + b\) and the product \(a \times b\) are uniquely defined real numbers. Two special real numbers are zero (0) and one (1). These operations satisfy:

Properties of real numbers:

Commutative: \(a + b = b + a\) \(ab = ba\)

Example: \(7 + 3 = 3 + 7 = 10\) \(5 \times 6 = 6 \times 5 = 30\)

Associative: \(a + (b + c) = (a + b) + c\) \(a(bc) = (ab)c\)

Example: \(3 + (4 + 7) = (3 + 4) + 7\) \(2 \times (5 \times 3) = (2 \times 5) \times 3\)
\(3 + 11 = 7 + 7 = 14\) \(2 \times 15 = 10 \times 3 = 30\)

Identity: \(a + 0 = 0 + a = a\) \(a \times 1 = 1 \times a = a\)

Example: \(8 + 0 = 0 + 8 = 8\) \(11 \times 1 = 1 \times 11 = 11\)

For each real number \(a\), there is a real number, denoted by \(-a\), called the negative of \(a\), for which

Inverse: \(a + (-a) = 0 = (-a) + a\)

Example: \(7 + (-7) = 0\)

Subtraction, denoted by \(a - b\), is defined as follows:
\(a - b = a + (-b)\)

For each \(a \neq 0\), there is a real number, denoted by \(\frac{1}{a}\) or \(1/a\) or \(a^{-1}\), called the reciprocal of \(a\), for which

Inverse: \(a \left(\frac{1}{a}\right) = 1 = (\frac{1}{a})a\)

Example: \(7 \times \frac{1}{7} = \frac{1}{7} \times 7 = 1\)

Division, denoted by \(a \div b\) or \(\frac{a}{b}\) or \(a/b\), where \(b \neq 0\), is defined as follows:
\(a + b = \frac{a}{b} = a/b = a\left(\frac{1}{b}\right) = ab^{-1}\)

Finally, there is a property which relates addition and multiplication:

Distributive: \(a(b + c) = ab + ac\) \((a + b)c = ac + bc\)

Example: \(-4 \times (3 + 2) = (-4 \times 3) + (-4 \times 2)\)
\(-4 \times 5 = -12 + (-8) = -20\)