Sequences and Series

Exercises

For problems 1-4, assume that the pattern continues and write out the general term of the sequence. Assume the terms are numbered beginning with $n = 1$.

1. \{1, 3, 5, 7, 9, \ldots\}
2. \left\{ \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32} \right\}
3. \left\{ \frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \frac{4}{25}, \frac{5}{36} \right\}
4. \left\{ \frac{2}{1}, \frac{4}{2}, \frac{8}{3}, \frac{16}{4}, \frac{32}{5} \right\}

For problems 5-8, make a table of the first eight terms of each series and then plot these terms.

5. $a_n = \frac{2n - 1}{n + 1}$
6. $\left\{ 3 + \frac{(-1)^n}{n} \right\}$
7. $b_n = 2 + (-1)^n$
8. $c_n = \frac{2^n}{n!}$

For problems 9-16 make a table of the of the first eight partial sums, plot the sequence of partial sums and use the plot to guess the sum of the series.

9. $\sum_{n=1}^{\infty} \frac{4}{3^n}$
10. $\sum_{n=1}^{\infty} \frac{4}{n^3}$
11. $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$
12. $\sum_{n=0}^{\infty} (-1)^n$
13. $\sum_{n=0}^{\infty} \frac{1}{n!}$ \quad Recall that $n$ factorial is $n! = n \cdot (n - 1) \cdot \cdots \cdot 2 \cdot 1$ and $0! = 1$
14. $\sum_{n=0}^{\infty} (-1)^n \frac{4}{2n - 1}$
15. $\sum_{n=1}^{\infty} \frac{1}{n}$
16. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

For problems 17-23, find the sum of the series or say why it diverges.

17. $\sum_{n=1}^{\infty} \frac{4}{3^n}$
18. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{4}{3^n}$
19. \(\frac{1}{3} - \frac{4}{9} + \frac{16}{27} - \frac{64}{81} + \frac{256}{243} - \ldots\)

20. \(1 + e^{-1} + e^{-2} + e^{-3} + \ldots\)

21. \(\sum_{n=1}^{\infty} (-1)^n \left(\frac{3}{2}\right)^n\)

22. \(\sum_{n=0}^{\infty} 2 \left(\frac{1}{\sqrt{2}}\right)^n\)

23. \(6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8} - \ldots\)

For problems 24-26 express each repeating decimal as a geometric series and write its sum as a ratio of integers.

24. \(0.\overline{444}\) (The bar above the digit or digits indicates the repeating portion.)

25. \(0.\overline{1212}\)

26. \(0.\overline{2714}\)

27. A ball is dropped from a height of 8 ft. On each bounce it rebounds to .8 of the height of the last bounce. Find the total distance (up and down) the ball travels.

28. A patient receives 5 mg. of a certain drug daily. If in one 24-hr day, the body eliminates 20% of this drug, estimate the total amount that would be present after extended treatment.