Solving Equations

Answers to Exercises

1. Substitute the value into the equation:

\[ \frac{0 - 1}{0 + 1} = \frac{0 + 1}{0 - 1} \quad ? \]
\[ \frac{-1}{1} = \frac{1}{-1} ? \]
\[ -1 = -1 \sqrt{1} \]

2. 

\[ -12 + 12 \left( \frac{3}{2} \right)^2 = 10 \left( \frac{3}{2} \right) ? \]
\[ -12 + 12 \left( \frac{9}{4} \right) = 10 \left( \frac{3}{2} \right) ? \]
\[ -12 + 27 = 15 ? \]
\[ 15 = 15 \sqrt{1} \]

3. 

\[ 25 - \sqrt{25} = 20 ? \]
\[ 25 - 5 = 20 ? \]
\[ 20 = 20 \sqrt{1} \]

4. 

\[ -12 + 12y = 10y \]
\[ -12 + 12y - 12y = 10y - 12y \]
\[ -12 = -2y \]
\[ \frac{-12}{-2} = \frac{-2y}{-2} \]
\[ y = 6 \]

5. 

\[ 2x + 1 = -2x + 1 \]
\[ 2x + 1 - 1 = -2x + 1 - 1 \]
\[ 2x = -2x \]
\[ 2x + 2x = -2x + 2x \]
\[ 4x = 0 \]
\[ \frac{4x}{4} = \frac{0}{4} \]
\[ x = 0 \]

6. Although this does not look like a linear equation at first, it will become one when we multiply both sides by the lowest common denominator:

\[ 12 \left( 1 + \frac{y}{3} - \frac{y}{4} \right) = 12 \left( y - \frac{5y}{6} \right) \]
\[ 12 + 4y - 3y = 12y - 10y \]
\[ y + 12 = 2y \]
\[ y + 12 - y = 2y - y \]
\[ 12 = y \]
7. 
\[ x^2 - 7x - 8 = 0 \]
\[(x - 8)(x + 1) = 0 \]
\[ x - 8 = 0 \text{ or } x + 1 = 0 \]
\[ x = 8 \text{ or } x = -1 \]

8. Remember to put all terms on one side of the equation:
\[ x^2 = -3x \]
\[ x^2 + 3x = 0 \]
\[ x(x + 3) = 0 \]
\[ x = 0 \text{ or } x + 3 = 0; x = -3 \]
NOTICE that if we had divided both sides by \( x \), we would have lost the solution \( x = 0 \):
\[ \frac{\frac{x^2}{x}}{x} = \frac{-3x}{x} \]
\[ x = -3 \text{ ONLY} \]

9. 
\[ 7x^2 + 16x = -9 \]
\[ 7x^2 + 16x + 9 = 0 \]
\[ (7x + 9)(x + 1) = 0 \]
\[ 7x + 9 = 0 \text{ or } x + 1 = 0 \]
\[ 7x = -9 \text{ or } x = -1 \]
\[ x = -\frac{9}{7} \text{ or } x = -1 \]

10. 
\[ 16x^2 - 18 = -2x \]
\[ 16x^2 + 2x - 18 = 0 \]
\[ 2(8x^2 + x - 9) = 0 \]
\[ 2(8x + 9)(x - 1) = 0 \]
\[ 8x + 9 = 0 \text{ or } x - 1 = 0 \]
\[ 8x = -9 \text{ or } x = 1 \]
\[ x = -\frac{9}{8} \text{ or } x = 1 \]

11. Although this equation is not quadratic, it can still be factored:
\[ x^3 - x^2 - 30x = 0 \]
\[ x(x^2 - x - 30) = 0 \]
\[ x(x - 6)(x + 5) = 0 \]
\[ x = 0 \text{ or } x - 6 = 0 \text{ or } x + 5 = 0 \]
\[ x = 0 \text{ or } x = 6 \text{ or } x = -5 \]

12. 
\[ x^2 + 6x - 10 = 0 \]
\[(x^2 + 6x + 9) - 10 - 9 = 0 \]
\[ x^2 + 6x + 9 = 19 \]
\[ (x + 3)^2 = 19 \]
\[ x + 3 = \pm \sqrt{19} \]
\[ x = -3 \pm \sqrt{19} \]
13. 

$$3y^2 + 12y + 8 = 0$$
$$3(y^2 + 4y) + 8 = 0$$
$$3(y^2 + 4y + 4) + 8 - 12 = 0 \text{ (don't forget to distribute the 3!)}$$
$$3(y + 2)^2 = 4$$
$$(y + 2)^2 = \frac{4}{3}$$
$$y + 2 = \pm \sqrt{\frac{4}{3}}$$
$$y + 2 = \pm \frac{2}{\sqrt{3}} \text{ (now rationalize)}$$
$$y + 2 = \pm \frac{2\sqrt{3}}{3}$$
$$y = -2 \pm \frac{2\sqrt{3}}{3} \text{ (common denominator)}$$
$$y = \frac{-6 \pm 2\sqrt{3}}{3}$$

14. 

$$a^2 - 3a = 9$$
$$a^2 - 3a + \left(\frac{3}{2}\right)^2 = 9 + \left(\frac{3}{2}\right)^2$$
$$\left(a - \frac{3}{2}\right)^2 = 9 + \frac{9}{4}$$
$$\left(a - \frac{3}{2}\right)^2 = \frac{45}{4}$$
$$a - \frac{3}{2} = \pm \frac{\sqrt{45}}{2}$$
$$a = \frac{3 \pm \sqrt{45}}{2} \text{ (simplify radical)}$$
$$a = \frac{3 \pm 3\sqrt{5}}{2}$$

15. 

$$2x^2 + 5 = x$$
$$2x^2 - x + 5 = 0$$
$$2\left(x^2 - \frac{1}{2}x\right) + 5 = 0$$
$$2\left(x^2 - \frac{1}{2}x + \left(\frac{1}{4}\right)^2\right) + 5 - 2\left(\frac{1}{4}\right)^2 = 0$$
$$2\left(x - \frac{1}{4}\right)^2 = -\frac{39}{8}$$
$$\left(x - \frac{1}{4}\right)^2 = -\frac{39}{16}$$
$$x - \frac{1}{4} = \pm \sqrt{-\frac{39}{16}}$$
$$x - \frac{1}{4} = \pm \frac{\sqrt{39}i}{4}$$
$$x = \frac{1}{4} \pm \frac{\sqrt{39}i}{4}$$
$$x = \frac{1 \pm \sqrt{39}i}{4}$$
16. Let $a = 1$, $b = 6$, and $c = -10$ in the quadratic formula:
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 + 40}}{2} = \frac{-6 \pm \sqrt{76}}{2} = \frac{-6 \pm 2\sqrt{19}}{2} = \frac{2(-3 \pm \sqrt{19})}{2} = -3 \pm \sqrt{19}
\]

17. Let $a = 3$, $b = 12$, and $c = 8$ in the quadratic formula:
\[
y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-12 \pm \sqrt{144 - 96}}{6} = \frac{-12 \pm \sqrt{48}}{6} = \frac{-12 \pm 4\sqrt{3}}{6} = \frac{2(-6 \pm 2\sqrt{3})}{6} = \frac{-6 \pm 2\sqrt{3}}{3}
\]

18. $2x^2 - x + 5 = 0$

Let $a = 2$, $b = -1$, and $c = 5$ in the quadratic formula:
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 40}}{4} = \frac{1 \pm \sqrt{-39}}{4} = \frac{1 \pm 39i}{4}
\]

19. This equation factors easily, although other methods could be used:
\[
x^2 - 13x + 30 = 0
\]
\[(x - 3)(x - 10) = 0
\]
\[x - 3 = 0 \text{ or } x - 10 = 0
\]
\[x = 3 \text{ or } x = 10
\]

20. We can easily solve this equation by isolating the variable, although other methods also work:
\[
3c^2 - 21 = 0
\]
\[
3c^2 = 21
\]
\[
c^2 = 7
\]
\[
c = \pm \sqrt{7}
\]
21. The quadratic formula is probably best for this (here \( a = -6, b = 5, \) and \( c = -3 \)):

\[
-6t^2 + 5t - 3 = 0
\]

\[
t = \frac{-5 \pm \sqrt{5^2 - 4(-6)(-3)}}{2(-6)}
\]

\[
= \frac{-5 \pm \sqrt{-47}}{-12}
\]

\[
= \frac{-5 \pm \sqrt{47i}}{-12}
\]

22. Again, we use quadratic formula with \( a = 1, b = -4, \) and \( c = 1 \):

\[
y = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}
\]

\[
= \frac{4 \pm \sqrt{12}}{2}
\]

\[
= \frac{4 \pm 2\sqrt{3}}{2}
\]

\[
= \frac{2(2 \pm \sqrt{3})}{2}
\]

\[
= 2 \pm \sqrt{3}
\]

23. We will use the quadratic formula to make an observation later. Here \( a = 3, b = -4, \) and \( c = -15 \):

\[
x = \frac{-(4) \pm \sqrt{(-4)^2 - 4(3)(-15)}}{2(3)}
\]

\[
= \frac{4 \pm \sqrt{196}}{6}
\]

\[
= \frac{4 \pm 14}{6}
\]

\[
x = \frac{18}{6} \text{ or } x = \frac{-10}{6}
\]

\[
x = 3 \text{ or } x = \frac{-5}{3}
\]

24. Set \( s = 0 \) and solve:

\[
-16t^2 + 30t + 4 = 0
\]

\[
t = \frac{-30 \pm \sqrt{30^2 - 4(-16)(4)}}{2(-16)}
\]

\[
= \frac{-30 \pm \sqrt{1156}}{-32}
\]

\[
= \frac{-30 \pm 34}{-32}
\]

\[
t = \frac{-64}{-32} \text{ or } t = \frac{4}{-32}
\]

\[
t = 2 \text{ or } t = \frac{1}{8} \text{ (not valid)}
\]

2 seconds after releasing the ball.

25. Let \( u = x^2 \). Then \( u^2 = x^4 \), so our equation becomes

\[
u^2 - 10u + 9 = 0
\]

\[
(u - 9)(u - 1) = 0
\]

\[
u = 9 \text{ or } u = 1
\]

Therefore,

\[
x^2 = 9 \text{ or } x^2 = 1
\]

\[
x = \pm 3 \text{ or } x = \pm 1
\]
26. Let \( u = x^{\frac{1}{3}} \). Then \( u^2 = x^{\frac{2}{3}} \), so our equation becomes
\[
\begin{align*}
  u^2 - 4u & = -3 \\
  u^2 - 4u + 3 & = 0 \\
  (u - 3)(u - 1) & = 0
\end{align*}
\]
\[ u = 3 \text{ or } u = 1 \]
\[ x^{\frac{1}{3}} = 3 \text{ or } x^{\frac{1}{3}} = 1 \]
\[ x = 3^3 \text{ or } x = 1^3 \]
\[ x = 27 \text{ or } x = 1 \]

27. Let \( u = b^2 \). Then \( u^2 = b^4 \), so our equation becomes
\[
\begin{align*}
  u^2 - 3u - 21 & = 0 \\
  (u - 7)(u + 4) & = 0
\end{align*}
\]
\[ u = 7 \text{ or } u = -4 \]
\[ b^2 = 7 \text{ or } b^2 = -4 \]
\[ b = \pm \sqrt{7} \text{ or } b = \pm 2i \]

28. Let \( u = y^{\frac{1}{4}} \). Then \( u^2 = y^{\frac{1}{2}} \), so our equation becomes
\[
\begin{align*}
  u^2 + 4u - 12 & = 0 \\
  (u + 6)(u - 2) & = 0
\end{align*}
\]
\[ u = -6 \text{ or } u = 2 \]
\[ y^{\frac{1}{4}} = -6 \text{ (no solution) or } y^{\frac{1}{4}} = 2 \]
\[ y = 2^4 = 16 \]

29. First, rewrite the equation in fraction form:
\[ \frac{-16}{x^2} - \frac{6}{x} + 1 = 0 \]
Now eliminate the fractions by multiplying by \( x^2 \):
\[ -16 - 6x + x^2 = 0 \]
\[ x^2 - 6x - 16 = 0 \]
\[ (x - 8)(x + 2) = 0 \]
\[ x = 8 \text{ or } x = -2 \]

30. LCD is 4x:
\[
\begin{align*}
  4x(1) - 4x\left(\frac{5}{x}\right) & = 4x\left(\frac{3}{4}\right) \\
  4x - 20 & = 3x \\
  x - 20 & = 0 \\
  x & = 20
\end{align*}
\]
Check:
\[ 1 - \frac{5}{20} = \frac{3}{4} \ ? \]
\[ 1 - \frac{1}{4} = \frac{3}{4} \ \checkmark \]
\[ x = 20 \text{ is the solution to the equation.} \]
31. LCD is \((y + 1)(y - 2)\) (or simply cross-multiply since the fractions are isolated on both sides):

\[
(y + 1)(y - 2) \left( \frac{2}{y + 1} \right) = (y + 1)(y - 2) \left( \frac{-1}{y - 2} \right)
\]

\[
2(y - 2) = 1(y + 1)
\]

\[
y = 5
\]

Check:

\[
\frac{2}{5 + 1} = \frac{1}{\frac{5}{2}} \quad ?
\]

\[
\frac{2}{3} = \frac{1}{\frac{5}{2}} \quad \checkmark
\]

Therefore, \(x = 5\) is the solution.

32. LCD is \((x + 1)(x - 1)\):

\[
(x + 1)(x - 1) \left( \frac{x - 1}{x + 1} \right) = (x + 1)(x - 1) \left( \frac{x + 1}{x - 1} \right)
\]

\[
(x - 1)(x + 1) = (x + 1)(x + 1) \text{ remember to FOIL}
\]

\[
x^2 - 2x + 1 = x^2 + 2x + 1
\]

\[
-4x = 0
\]

\[
x = 0
\]

Check:

\[
\frac{0 - 1}{0 + 1} = \frac{0 + 1}{0 - 1} \quad ?
\]

\[
-1 = -1 \quad \checkmark
\]

Therefore, \(x = 0\) is the solution.

33. LCD is \(2(a - 1)\):

\[
2(a - 1) \left( \frac{a - 1}{2} \right) = 2(a - 1) \left( \frac{a + 11}{a - 1} \right)
\]

\[
(a - 1)(a - 1) = 2(a + 11)
\]

\[
a^2 - 2a + 1 = 2a + 22 \text{ quadratic equation}
\]

\[
a^2 - 4a - 21 = 0
\]

\[
(a - 7)(a + 3) = 0
\]

\[
a = 7 \quad \text{or} \quad a = -3
\]

Check:

\[
\frac{7 - 1}{2} = \frac{7 + 11}{7 - 1} \quad ?
\]

\[
\frac{6}{2} = \frac{18}{6} \quad ?
\]

\[
2 = 3 \quad \checkmark
\]

Therefore, \(x = 7\) and \(x = -3\) are the solutions.

34. Factor first:

\[
\frac{y - 3}{y^2 - 4y} = \frac{2}{y^2 - 16}
\]

\[
\frac{y - 3}{y(y - 4)} = \frac{2}{(y + 4)(y - 4)}
\]

The LCD is \(y(y - 4)(y + 4)\):
\[
y(y - 4)(y + 4) - \frac{y - 3}{y(y - 4)} = y(y - 4)(y + 4) - \frac{2}{(y + 4)(y - 4)}
\]
\[
(y + 4)(y - 3) = 2y
\]
\[
y^2 + y - 12 = 2y
\]
\[
y^2 - y - 12 = 0
\]
\[
(y - 4)(y + 3) = 0
\]
\[
y = 4 \text{ or } y = -3
\]

Check:
\[
\frac{-3 - 3}{(-3)^2 - 4(-3)} = \frac{2}{(-3)^2 - 16} \quad \frac{4 - 3}{4^2 - 4(4)} = \frac{2}{(4)^2 - 16}
\]
\[
\frac{-6}{9 + 12} = \frac{2}{9 - 16} \quad \frac{1}{16 - 16} = \frac{2}{16 - 16}
\]
\[
\frac{-6}{21} = -\frac{2}{7} \quad \sqrt{10} = \frac{2}{0} \quad \text{Undefined}
\]

Therefore, the only solution is \(y = -3\).

35. The LCD is \((x - 4)(x + 5)\):
\[
(x - 4)(x + 5) \left(\frac{-2}{x + 4}\right) + (x - 4)(x + 5) \left(\frac{3x}{x + 5}\right) = 0(x - 4)(x + 5)
\]
\[
2(x + 5) + 3x(x - 4) = 0
\]
\[
2x + 10 + 3x^2 - 12x = 0
\]
\[
3x^2 - 10x + 10 = 0 \quad \text{use quadratic formula}
\]
\[
x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(10)}}{2(3)}
\]
\[
x = \frac{10 \pm \sqrt{100}}{6}
\]
\[
x = \frac{10 \pm 10}{6}
\]
\[
x = \frac{2(5 \pm i\sqrt{5})}{6}
\]
\[
x = \frac{5 \pm i\sqrt{5}}{3}
\]

Since this is a complex solution, it will not be an extraneous solution.

36. First, see if factoring helps. Note that since this is multiplication, the LCD is not necessary!
\[
x^2 - 4x + 4 \cdot x^2 + 5x + 6 = 0
\]
\[
(x - 2)^2 \cdot (x + 2)(x + 3) = 0
\]
\[
(x - 2)(x + 3) = 0
\]
\[
x = 2 \text{ or } x = -3
\]

Check:
\[
\frac{(2)^2 - 4(2) + 4}{(2) + 2} \cdot \frac{(2)^2 + 5(2) + 6}{(2) - 2} = 0 ? \quad \frac{(-3)^2 - 4(-3) + 4}{(-3) + 2} \cdot \frac{(-3)^2 + 5(-3) + 6}{(-3) - 2} = 0
\]
\[
\frac{0}{4} \cdot \frac{20}{0} = 0 \quad \text{UNDEFINED} \quad \frac{25}{-1} \cdot \frac{0}{-5} = 0 ?
\]
\[
0 = 0 \checkmark
\]

Therefore, the only solution is \(x = -3\). 
37. Suppose \(2 + \frac{5}{x-4} = \frac{x+1}{x-4}\) has a solution. Then it must be true that
\[
2 + \frac{5}{x-4} = \frac{x+1}{x-4} \\
\frac{2(x-4) + 5}{x-4} = \frac{x+1}{x-4} \\
\frac{2x-8+5}{x-4} = \frac{x+1}{x-4}
\]
The only way two fractions with the same denominator can be equal is if the numerators are equal. Thus, we must have
\[
2x - 3 = x + 1 \\
x - 3 = 1 \\
x = 4.
\]
Thus, if we have a solution it must equal 4, but 4 is not acceptable, since we cannot divide by 0.

38. First, isolate the radical, then square both sides:
\[
3 + 2\sqrt{x+1} = 7 \\
2\sqrt{x+1} = 4 \\
\sqrt{x+1} = 2 \\
x + 1 = 4 \\
x = 3
\]
Check:
\[
3 + 2\sqrt{3+1} = 7 \, ? \\
3 + 2\sqrt{4} = 7 \, ? \\
3 + 4 = 7 \, \sqrt{ }
\]
Therefore, \(x = 3\) is the solution.

39. Again, isolate the radical and square both sides:
\[
3\sqrt{x+7} - 4 = 8 \\
3\sqrt{x+7} = 12 \\
\sqrt{x+7} = 4 \\
x + 7 = 16 \\
x = 9
\]
Check:
\[
3\sqrt{9+7} - 4 = 8 \, ? \\
3\sqrt{16} - 4 = 8 \, ? \\
12 - 4 = 8 \, \sqrt{ }
\]
Therefore, \(x = 9\) is the solution.

40. Isolate the radical and square both sides. This time, you have a quadratic equation to solve after squaring:
\[
\sqrt{y-4} - y = -4 \\
\sqrt{y-4} = y - 4 \\
y - 4 = y^2 - 8y + 16 \, \text{(remember FOIL)} \\
0 = y^2 - 9y + 20 \\
0 = (y - 5)(y - 4) \\
y = 5 \text{ or } y = 4
Check:
\[
\sqrt{5 - 4} - 5 = -4 \ ? \quad \sqrt{4 - 4} - 4 = -4 \ ? \\
\sqrt{1 - 5} = -4 \ ? \quad \sqrt{0 - 4} = -4 \ ? \\
1 - 5 = -4 \sqrt{0} \quad 0 - 4 = -4 \sqrt{1}
\]
Therefore, the solutions are \( x = 5 \) and \( x = 4 \).

41. As in the previous problem, isolate (already done), square and solve the resulting quadratic equation:
\[
\sqrt{x + 5} = x - 1 \\
x + 5 = x^2 - 2x + 1 \\
0 = x^2 - 3x - 4 \\
0 = (x - 4)(x + 1) \\
x = 4 \text{ or } x = -1
\]
Check:
\[
\sqrt{4 + 5} = 4 - 1 \ ? \quad \sqrt{-1 + 5} = -1 - 1 \ ? \\
\sqrt{9} = 3 \sqrt{4} = -2 \text{ X}
\]
Therefore, the only solution is \( x = 4 \).

42. Isolate the radical, square, and solve the resulting quadratic equation:
\[
z - \sqrt{z} = 20 \\
z - 20 = \sqrt{z} \\
z^2 - 40z + 400 = z \\
z^2 - 41z + 400 = 0 \\
z = \frac{41 \pm \sqrt{(-41)^2 - 4(1)(400)}}{2} \\
z = \frac{41 \pm 9}{2} \\
z = \frac{41 + 9}{2} = 25 \text{ or } z = \frac{41 - 9}{2} = 16
\]
Check:
\[
25 - \sqrt{25} = 20 \ ? \quad 16 - \sqrt{16} = 20 \ ? \\
25 - 5 = 20 \sqrt{16} = 20 \text{ X}
\]
Therefore, the only solution is \( x = 25 \).

43. This time we must isolate one radical at a time:
\[
\sqrt{y - 5} = \sqrt{y + 4} - 1 \\
y - 5 = (y + 4) - 2\sqrt{y + 4} + 1 \text{ (FOIL!!!)} \\
y - 5 = y + 5 - 2\sqrt{y + 4} \\
-10 = -2\sqrt{y + 4} \\
5 = \sqrt{y + 4} \\
25 = y + 4 \\
y = 21
Check:
\[
\sqrt{21 - 5} = \sqrt{21 + 4} - 1 \quad ? \\
\sqrt{16} = \sqrt{25 - 1} \quad ? \\
4 = 5 - 1 \sqrt{\ }
\]
Therefore, the solution is \( x = 21 \).

44. The absolute value leads to 2 equations:
\[
|x - 9| = 3 \\
x - 9 = 3 \quad -(x - 9) = 3 \\
x = 12 \quad -x + 9 = 3 \\
-x = -6 \\
x = 6
\]
Therefore, the solutions are \( x = 12 \) and \( x = 6 \). It is advisable to check that both solutions are valid.

45. Again write 2 equations (note the \(-12\) does not change sign since it is not in the absolute value):
\[
|6b - 3| - 12 = 0 \\
6b - 3 - 12 = 0 \quad -(6b - 3) - 12 = 0 \\
6b - 15 = 0 \quad -6b + 3 - 12 = 0 \\
6b = 15 \quad -6b - 9 = 0 \\
b = \frac{5}{2} \quad -6b = 9 \\
\quad \quad b = -\frac{3}{2}
\]
Therefore, the solutions are \( b = \frac{5}{2} \) and \( b = -\frac{3}{2} \). It is advisable to check that both solutions are valid.

46. Write the 2 equations:
\[
\left| \frac{2}{5}x - 5 \right| = 3 \\
\frac{2}{5}x - 5 = 3 \quad -\left( \frac{2}{5}x - 5 \right) = 3 \\
\frac{2}{5}x = 8 \quad -\frac{2}{5}x + 5 = 3 \\
x = 20 \quad -\frac{2}{5}x = -2 \\
\quad \quad x = 5
\]
Therefore, the solutions are \( x = 20 \) and \( x = 5 \). It is advisable to check that both solutions are valid.

47. Although both absolute values change sign, you only have to change one side to solve (the other leads to the same equations):
\[
|y + 6| = |2y - 2| \\
y + 6 = 2y - 2 \quad y + 6 = -(2y - 2) \\
-y = -8 \quad y + 6 = -2y + 2 \\
y = 8 \quad 3y = -4 \\
\quad \quad y = -\frac{4}{3}
\]
Therefore, the solutions are \( y = 8 \) and \( y = -\frac{4}{3} \). It is advisable to check that both solutions are valid.
48. It is recommended to isolate the absolute value before writing the 2 equations to avoid sign errors:

\[3|2x + 1| - 4 = 6\]
\[3|2x + 1| = 10\]
\[|2x + 1| = \frac{10}{3}\]

\[2x + 1 = \frac{10}{3}, \quad -(2x + 1) = \frac{10}{3}\]
\[2x = \frac{7}{3}, \quad -2x - 1 = \frac{10}{3}\]
\[x = \frac{7}{6}, \quad -2x = \frac{13}{3}\]
\[x = -\frac{13}{6}\]

Therefore the solutions are \(x = \frac{7}{6}\) and \(x = -\frac{13}{6}\). It is advisable to check that both solutions are valid.

49. Again isolate the absolute value:

\[5|x - 1| - 6 = -9\]
\[5|x - 1| = -3\]
\[|x - 1| = -\frac{3}{5}\]

Recall that absolute value represents the distance from zero on the number line. Therefore, it cannot be negative, so this equation has no solutions.

50. The equation is linear in \(h\), so we solve by isolating the variable:

\[V = \pi r^2 h\] divide by \(\pi r^2\)

\[\frac{V}{\pi r^2} = h\]

51. The equation is now quadratic in \(r\). This quadratic can be solved by isolating \(r^2\) and taking the square root:

\[V = \pi r^2 h\] divide by \(\pi h\)

\[\frac{V}{\pi h} = r^2\]

\[r = \pm \sqrt{\frac{V}{\pi h}}\]

(In general, \(r\) represents the radius of a cylinder, so only the positive square root would make practical sense).
52. The LCD is \( V_2P_1 \), so multiply both sides:

\[
\begin{align*}
(V_2P_1) \frac{V_1}{V_2} &= \frac{P_2}{P_1} (V_2P_1) \\
P_1V_1 &= P_2V_2 \\
V_1 &= \frac{P_2V_2}{P_1}
\end{align*}
\]

53. As in the previous problem, multiply by the LCD \( V_2P_1 \):

\[
\begin{align*}
(V_2P_1) \frac{V_1}{V_2} &= \frac{P_2}{P_1} (V_2P_1) \\
P_1V_1 &= P_2V_2 \\
P_1 &= \frac{P_2V_2}{V_1}
\end{align*}
\]

54. The equation is linear in \( b_2 \), so solve by isolating the variable:

\[
\begin{align*}
A &= \frac{1}{2} h (b_1 + b_2) \\
A &= \frac{1}{2} hb_1 + \frac{1}{2} hb_2 \\
A - \frac{1}{2} hb_2 &= \frac{1}{2} hb_1 \\
2A - hb_2 &= hb_1 \\
\frac{2A - hb_2}{h} &= b_1
\end{align*}
\]

Other methods of solution are possible.

55. This equation is rational in \( F \), so multiply by the LCD \( F + U \):

\[
\begin{align*}
(F + U)P &= \frac{F}{F + U} (F + U) \\
PF + PU &= F \quad \text{isolate} \ F \text{ terms} \\
PF - F &= PU \\
F(P - 1) &= PU \\
F &= \frac{PU}{P - 1}
\end{align*}
\]

56. Square both sides to eliminate the radical:

\[
\begin{align*}
v &= \sqrt{2as} \\
v^2 &= 2as \\
s &= \frac{v^2}{2a}
\end{align*}
\]